O K L A H O M A   S T A T E   U N I V E R S I T Y

SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECEN 5713 Linear Systems
Spring 2001
Midterm Exam #2

DO ALL FIVE PROBLEMS

Name : ______________________________

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Problem 1:

If u_1 and u_2 are linearly independent of each other, and $w_1 = au_1 + bu_2$, $w_2 = cu_1 + du_2$, please derive the relationship among $\{a, b, c, d\}$ such that w_1 and w_2 are linearly independent of each other.

Problem 2:

Consider the linear operator

$$A = \begin{bmatrix} 1 & 2 & -1 & 0 \\ 2 & 4 & -2 & 0 \\ 1 & 2 & -1 & 0 \end{bmatrix},$$

determine its rank and nullity, then find a basis for the range space and the null space of the linear operator, A, respectively ?

Problem 3:

Consider the subspace of $\Re^4$ consisting of all 4×1 column vector $x = [x_1 \quad x_2 \quad x_3 \quad x_4]^T$ with $x_1 + x_2 + x_3 = 0$. Extend the following set to form a basis for the space:

$$\begin{bmatrix} 1 \\ -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}.$$

<u>Problem 4</u>:

Extend the set

$$\begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -1 \\ 4 \end{bmatrix}$$

to form a basis in $\left(\Re^4, \Re\right)$.

Problem 5:

Let

$$V^{\perp} = Span\left(\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \begin{bmatrix} -5 & 1 \\ 1 & 5 \end{bmatrix}, \begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix}\right),$$

determine the original space, V. For $x = \begin{bmatrix} 0 & 3 \\ 3 & 0 \end{bmatrix}$, find its direct sum representation of $x = x_1 \oplus x_2$, such that $x_1 \in V$, and $x_2 \in V^{\perp}$ (I.e., the direct sum of spaces V and $V^{\perp}$ is the set of all 2×2 matrics with real coefficients).